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Pseudospherical 3- Planes In $\mathbb{R}^5$ and Evolution Equations Of Type $u_{tt} = \psi\left(u, u_x, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, \dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}, u_t\right)$

M.F. El-Sabbagh[1], K.R. Abdo[2]
[1](Mathematics Department/ Faculty of Science, Minia University, Egypt)
[2](Mathematics Department/ Faculty of Science ,fayoum University)

Abstract: *In this paper, evolution equations with two or more spatial variables, which may describe pseudospherical planes in higher dimensions, are considered. Necessary and sufficient conditions for equations of type* $u_{tt} = \psi\left(u, u_x, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, \dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}, u_t\right)$
to describe a 3-dimensionalpseudospherical plane of R^5 *are given . Such equations are characterized.*
Keywords: *Evolution equations, Pseudospherical surfaces, Riemannian manifold, Solitons and* differential equations.

I. Introduction

It has been observed that exactly solvable nonlinear differential equations with two independent variables are obtained as compatibility conditions for linear systems. Moreover, obtaining a spectral linear problem associated with a nonlinear equation[14-16] has been useful in order to solve the initial value problem by the inverse scattering method, [1]. In [2,13] the notion of a differential equation which describe pseudospherical surfaces (surfaces with constant negative Gaussian curvature inR^5) was given . Studies are made in this direction concerning non linear evolution equations of type

$$u_{xt} = \psi\left(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}\right) \text{ in [2,11]}$$

$$u_t = \psi\left(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}\right) \text{ in [1,10]}$$

And $\quad u_{tt} = \psi(u, u_x, , u_{xx}, u_t) \quad$ in [8].

Then, we [6,7] generalized these studies to evolution equations with three independent variables which are related to pseudospherical planes (P.S.P) in R^5 (3-dim planes of R^5 with constant negative sectional curvature) where equations of types

$$u_{xt} = \psi\left(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, u_{yy}, \dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}\right) \quad (1)$$

and

$$u_t = \psi\left(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, u_{yy}, \dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}\right) \quad (2)$$

Are studied . Here we provide a similar study for another class of evolution equations with two spatial variables plus the time variable which have the from

$$u_{tt} = \psi\left(u, u_x, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, \dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}, u_t\right) \quad (3)$$

II. Basic notations and Preliminaries

A differential equation E-for a real function $u(x, y, t)$ describes a 3-dimensional pseudospherical plane in R^5(simply P.S.P.) if it is the necessary and sufficient condition for the existence of differentiable functions $f_{\alpha i}, 1 \le \alpha \le 6$ and$1 \le i \le 3$, depending on u and its derivatives, such that the 1-forms

$$\omega_\alpha = f_{\alpha 1} dx + f_{\alpha 2} dy + f_{\alpha 3} dt$$

satisfy the structure equations of a 3-plane of constant sectional curvature -1 in R^5 i.e. equations (4).

$$\left.\begin{aligned} d\omega_1 &= \omega_4 \wedge \omega_2 + \omega_5 \wedge \omega_3 \\ d\omega_2 &= -\omega_4 \wedge \omega_1 + \omega_6 \wedge \omega_3 \\ d\omega_3 &= -\omega_5 \wedge \omega_1 - \omega_6 \wedge \omega_2 \\ d\omega_4 &= \omega_1 \wedge \omega_2 \\ d\omega_5 &= \omega_1 \wedge \omega_3 \\ d\omega_6 &= \omega_2 \wedge \omega_3 \end{aligned}\right\} \quad (4)$$