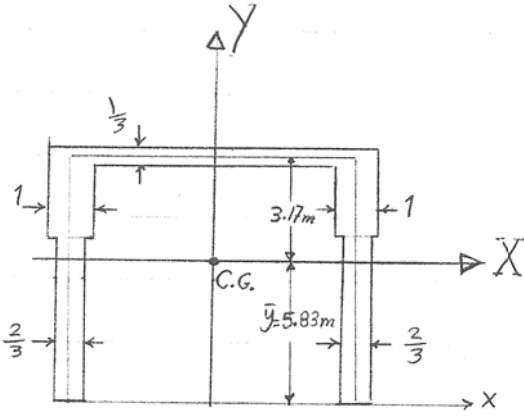
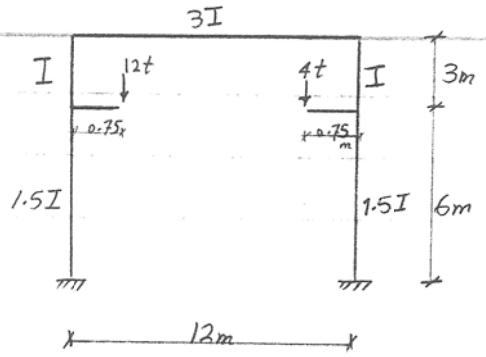


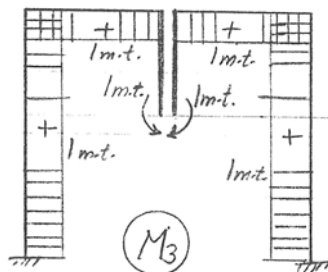
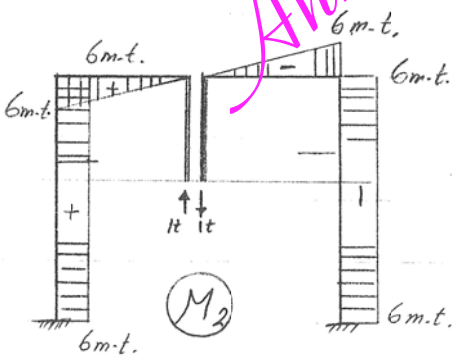
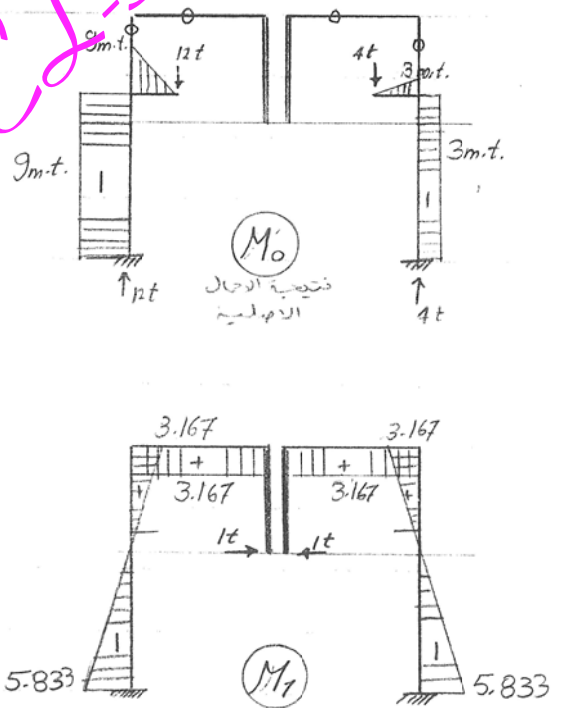
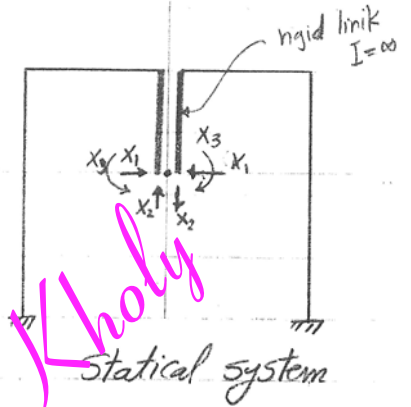
Example 1



$$\bar{y} = \frac{\sum A\bar{y}}{\sum A}$$

$$= \frac{\frac{1}{3} \times 12 \times 9 + 2 \times 1 \times 3 \times 7.5 + 2 \times \frac{2}{3} \times 6 \times 3}{\frac{1}{3} \times 12 + 2 \times 1 \times 3 + 2 \times \frac{2}{3} \times 6}$$

$$\bar{y} = 5.833 \text{ m}$$



$$\delta_{10} = \int \frac{M_1 M_0}{EI} dl$$

$$= \frac{1}{EI} \left[\frac{6}{6(1.5)} (50.99) + \frac{6}{6(1.5)} (152.98) \right]$$

$$\underline{\underline{\delta_{10} = 135.96/EI}}$$

$$\delta_{20} = \int \frac{M_2 M_0}{EI} dl$$

$$= \frac{1}{EI} \left[\frac{6}{6(1.5)} (108) + \frac{6}{6(1.5)} (-324) \right]$$

$$\underline{\underline{\delta_{20} = -144/EI}}$$

$$\delta_{30} = \int \frac{M_3 M_0}{EI} dl$$

$$= \frac{1}{EI} \left[\frac{6}{6(1.5)} (-18) + \frac{6}{6(1.5)} (-54) \right]$$

$$\underline{\underline{\delta_{30} = -48/EI}}$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dl = \frac{1}{EI} \left[\frac{2 \times 6}{6(1.5)} (66.16) + \frac{2 \times 3}{6} (21.17) + \frac{12}{6(3)} (60.17) \right]$$

$$\underline{\underline{\delta_{11} = 149.5/EI}}$$

$$\delta_{22} = \int \frac{M_2 M_2}{EI} dl = \frac{1}{EI} \left[\frac{2 \times 6}{6(1.5)} (216) + \frac{2 \times 3}{6} (216) + \frac{12}{6(3)} (72) \right]$$

$$\underline{\underline{\delta_{22} = 552/EI}}$$

$$\delta_{33} = \int \frac{M_3 M_3}{EI} dl = \frac{1}{EI} \left[\frac{2 \times 6}{6(1.5)} (6) + \frac{2 \times 3}{6} (6) + \frac{12}{6(3)} (6) \right]$$

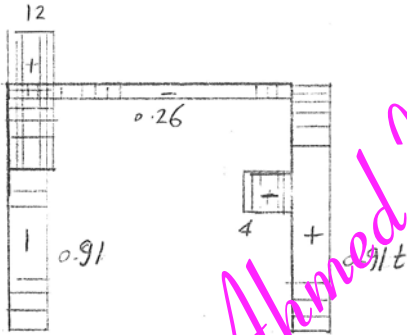
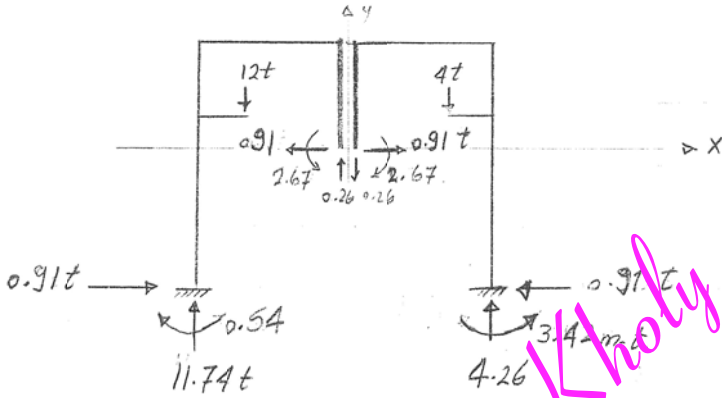
$$\underline{\underline{\delta_{33} = 18/EI}}$$

X & Y are symmetry axes $\rightarrow \delta_{12} = 0$

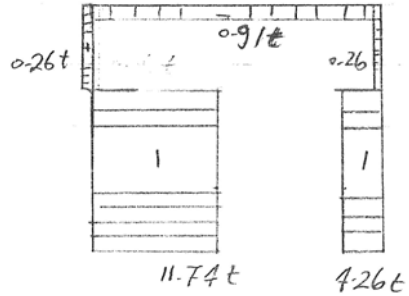
$$\delta_{10} + X_1 \delta_{11} = 0 \rightarrow X_1 = -0.91 t$$

$$\delta_{20} + X_2 \delta_{22} = 0 \rightarrow X_2 = 0.26 t$$

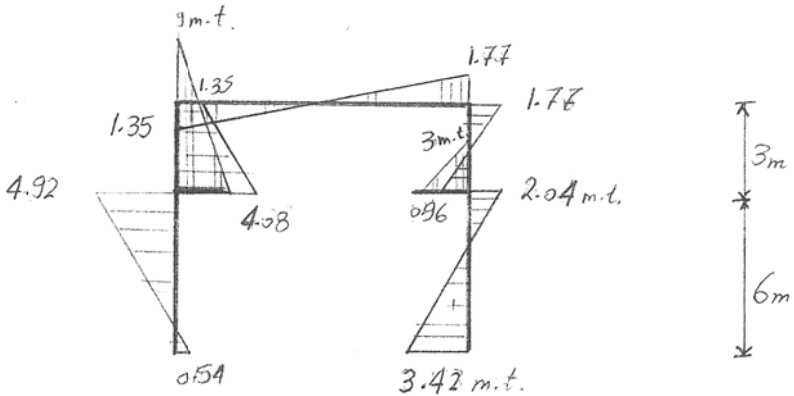
$$\delta_{30} + X_3 \delta_{33} = 0 \rightarrow X_3 = 2.67 \text{ m.t.}$$



S.F.D.

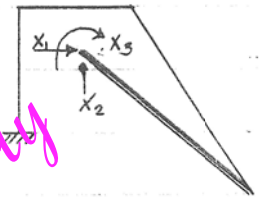
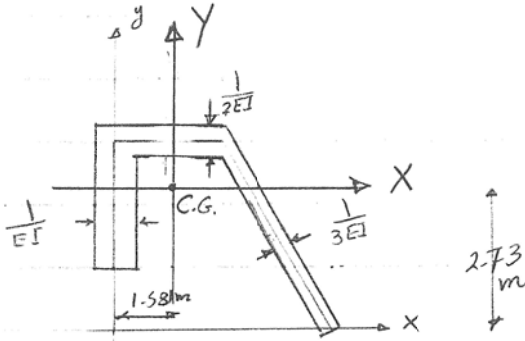
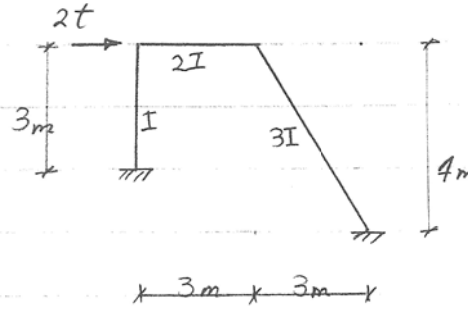


N.F.D.



B.M.D.

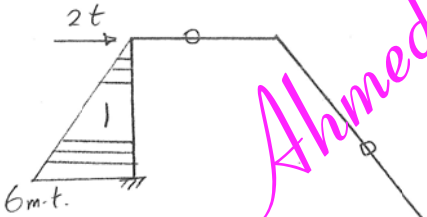
Example 2 Q1 Sheet



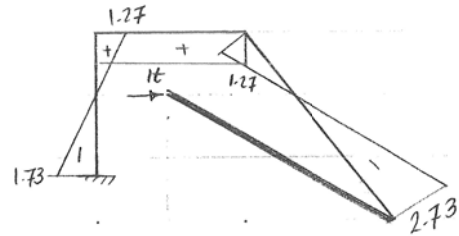
Static system

$$\bar{x} = \frac{\sum A \bar{x}}{\sum A} = \frac{(3 \times 1)(0) + (3 \times \frac{1}{2})(1.5) + (5 \times \frac{1}{3})(4.5)}{(3 \times 1) + (3 \times \frac{1}{2}) + (5 \times \frac{1}{3})} = 1.581 \text{ m}$$

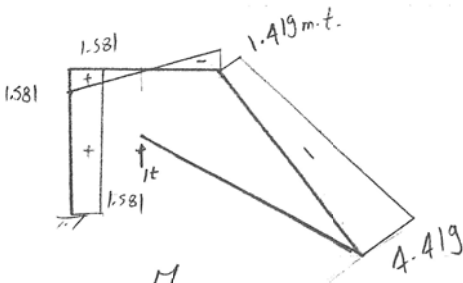
$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = \frac{(3 \times 1)(2.5) + (3 \times \frac{1}{2})(4) + (5 \times \frac{1}{3})(2)}{(3 \times 1) + (3 \times \frac{1}{2}) + (5 \times \frac{1}{3})} = 2.730 \text{ m}$$



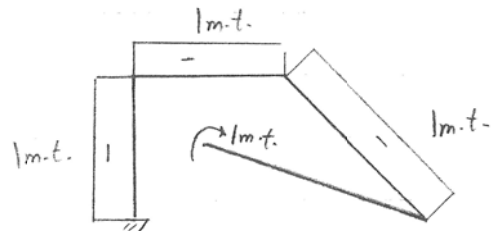
M₀



M₁



M₂



M₃

$$\delta_{10} = 6.57/EI, \quad \delta_{20} = -14.229/EI, \quad \delta_{30} = 9/EI$$

$$\delta_{11} = 7.938/EI, \quad \delta_{22} = 24.084/EI, \quad \delta_{33} = 6.167/EI$$

X & Y are not axes of symmetry = $I_{xy} \neq 0$

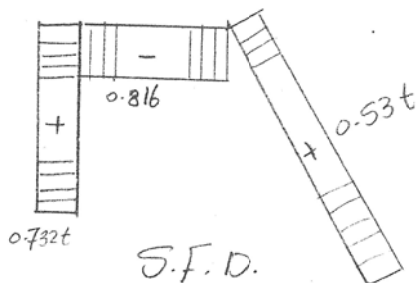
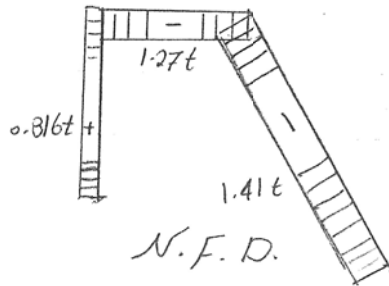
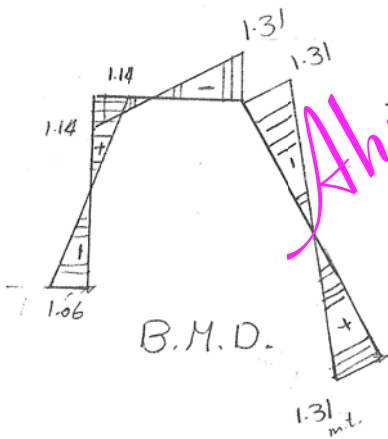
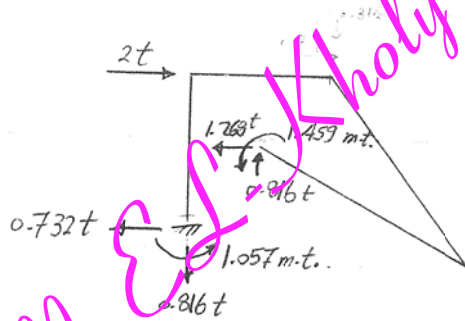
$$\int \frac{M_1 M_2}{EI} dl \neq 0$$

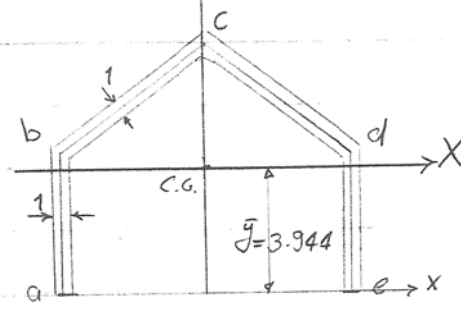
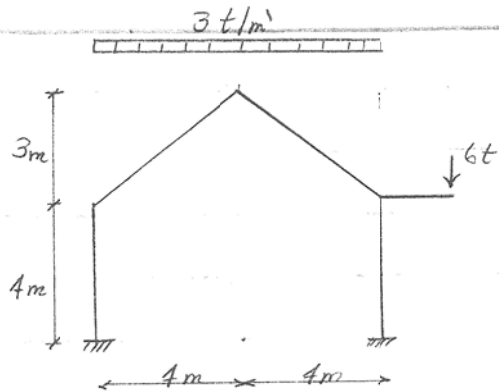
$$\delta_{10} + X_1 \delta_{11} + X_2 \delta_{12} = 0$$

$$\delta_{20} + X_1 \delta_{21} + X_2 \delta_{22} = 0$$

$$\delta_{30} + X_3 \delta_{33} = 0$$

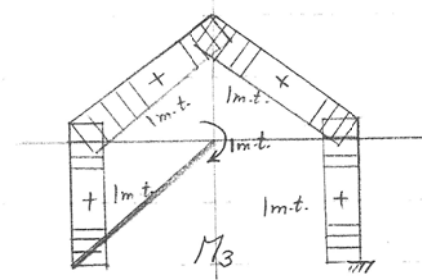
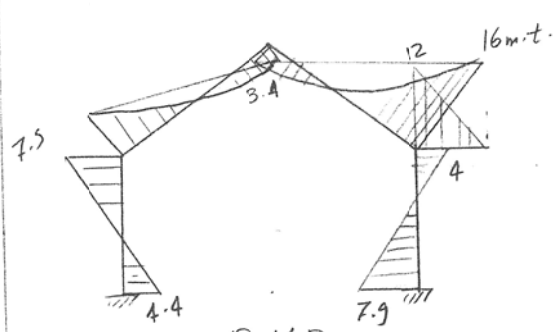
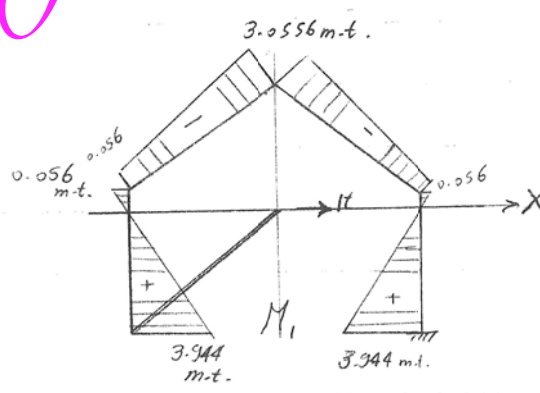
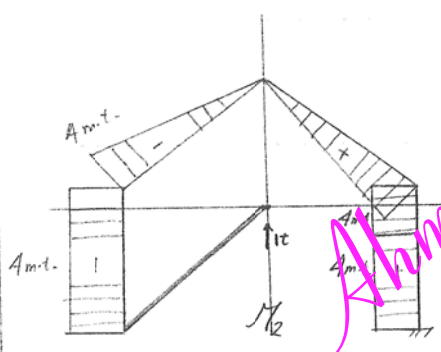
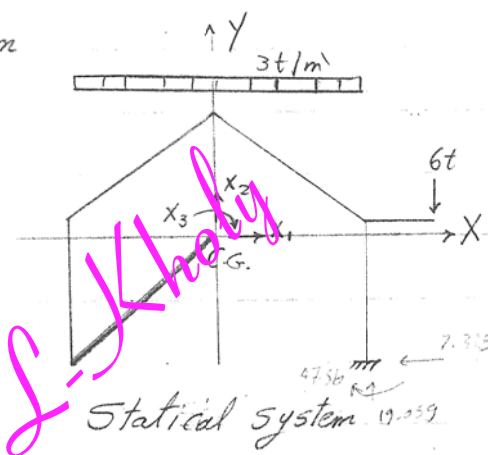
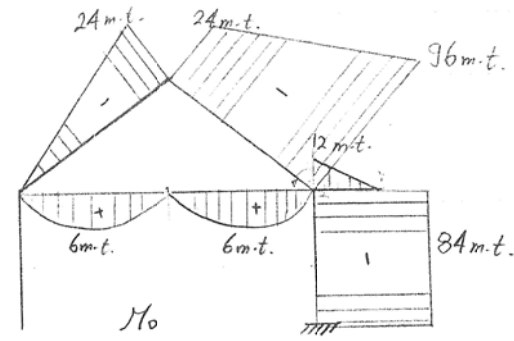
$$\begin{cases} 6.57 + 7.938X_1 + 4.282X_2 = 0 \\ -14.229 + 4.282X_1 + 24.084X_2 = 0 \\ 9 + 6.167X_3 = 0 \end{cases} \begin{cases} X_1 = -1.268 t \\ X_2 = 0.816 t \\ X_3 = -1.459 \text{ m.t.} \end{cases}$$





$$\bar{y} = \frac{\sum A \bar{y}}{\sum A}$$

$$\bar{y} = \frac{2(4 \times 1) \times 2 + 2(5 \times 1) \times 5.5}{2(4 \times 1) + 2(5 \times 1)} = 3.944 \text{ m}$$



B.M.D.

Using Elastic Center Method

$$\begin{aligned} \delta_{10} &= -215.556, \delta_{20} = -1984, \delta_{30} = -656 \\ \delta_{11} &= 72.6111, \delta_{22} = 181.333, \delta_{33} = 18 \\ X_1 &= 2.969, X_2 = 10.941, X_3 = 36.444 \\ M_{final} &= M_0 + X_1 M_1 + X_2 M_2 + X_3 M_3 \end{aligned}$$