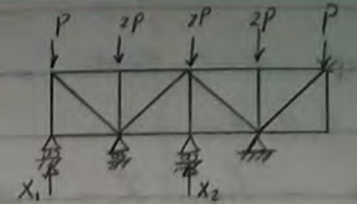
Matrix Force Method

1- Trusses

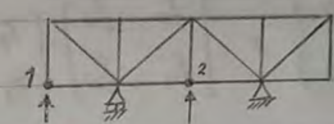
1



$J = 10$, $m = 17$, $r = 5$

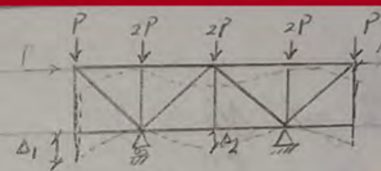
$n = 17 + 5 - 2 \times 10 \longrightarrow \underline{n = 2}$

Coordinates & primary system



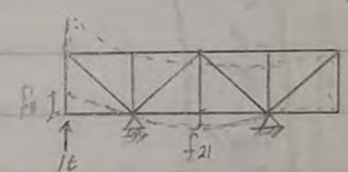
2

Δ due to external loads



$N_p = \begin{Bmatrix} - \\ - \\ - \end{Bmatrix}_{m\times 1}$ (17×1)

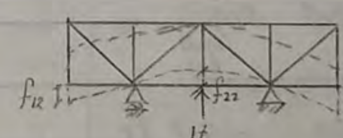
+



$\times X_1$

$N_q{}_{m\times n} = \begin{bmatrix} - & - \\ - & - \\ - & - \end{bmatrix}$ (17×2), columns $X_1 = 1$, $X_2 = 1$

+



$\times X_2$

on primary sys

3

$\Delta_1 + f_{11} X_1 + f_{12} X_2 = 0$

$\Delta_2 + f_{21} X_1 + f_{22} X_2 = 0$

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$\underline{n=2}$

$$\underset{(n\times1)}{\Delta} + \underset{(n\times n)}{F} \; \underset{(n\times1)}{X} = \underset{(n\times1)}{0}$$

4

Compatibility Condition

$$\underset{(n\times1)}{\Delta} + \underset{(n\times n)}{F} \; \underset{(n\times1)}{X} = \underset{(n\times1)}{0} \quad \text{(1)}$$

Where

$$\underset{(n\times1)}{\Delta} = \underset{(n\times m)}{N_q^T} \cdot \underset{(m\times m)}{F_u} \cdot \underset{(m\times1)}{N_p} \rightarrow \text{(2)}$$

and the flexibility matrix $F_{(n\times n)}$ is

$$\underset{(n\times n)}{F} = \underset{(n\times m)}{N_q^T} \cdot \underset{(m\times m)}{F_u} \cdot \underset{(m\times n)}{N_q} \rightarrow \text{(3)}$$

in (2) $\xrightarrow{get}$ $\Delta_{(n\times1)}$ ✓

in (3) $\xrightarrow{get}$ $F_{(n\times n)}$ ✓

in (1) $\xrightarrow{get}$ $X_{(n\times1)}$

&

$$\underset{(m\times1)}{N} = \underset{(m\times1)}{N_p} + \underset{(m\times n)}{N_q} \; \underset{(n\times1)}{X}$$

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5

- N_q = n.f in truss members due to unit load
- N_p = normal force in truss members due to applied loads
- Δ = displacement due to external loads (on the primary system)
- F = flexibility matrix
- F_{ij} = displacement at point i due to unit load at point j

X = unknowns (forces required $X_1, X_2, \ldots X_n$)

N = Normal force "required"

n = degree of indeterminacy → Statically

m = Number of members in truss

6

$$\underset{(m\times m)}{F_u} = \begin{bmatrix} \left(\frac{L}{EA}\right)_1 & 0 & \cdots & 0 \\ 0 & \left(\frac{L}{EA}\right)_2 & & \vdots \\ 0 & & \ddots & \\ 0 & 0 & \cdots & \left(\frac{L}{EA}\right)_m \end{bmatrix}_{m\times m}$$

= flexibility matrix for unassembled structure

EX-01 Truss; from 2016-2017 Final Exam - Modal Answer

2/5

L4

(23)

$EA = 10500\,t$

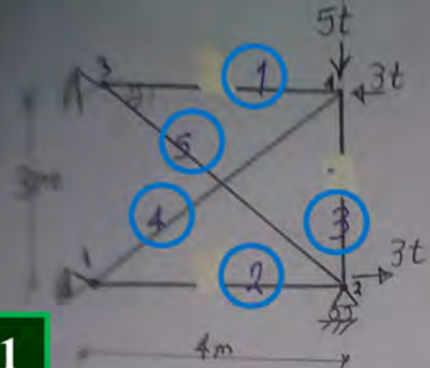
$\theta = 36.8699^\circ$

Final normal forces in the members

10

$$N = N_p + N_q X = \begin{Bmatrix} -1.2778 \\ 1.9841 \\ -3.7083 \\ -2.1528 \\ 1.2698 \end{Bmatrix} t$$

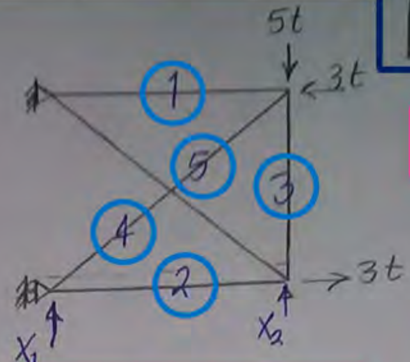
1



problem

$m + r = 5 + 5 = 10$

2

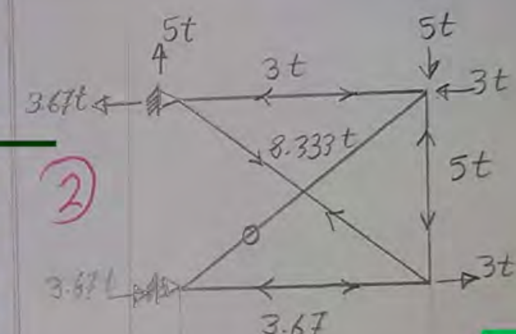


$2J = 8$

primary system

3

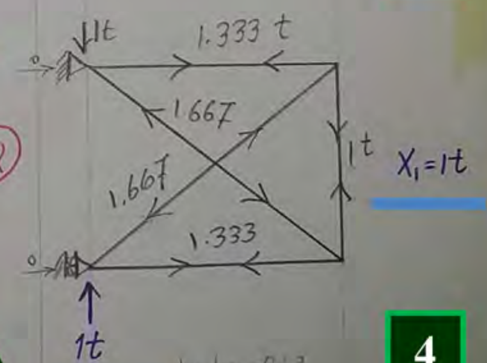
$$N_p = \begin{Bmatrix} -3 \\ -3.67 \\ -5 \\ 0 \\ 8.333 \end{Bmatrix} t \quad (2)$$



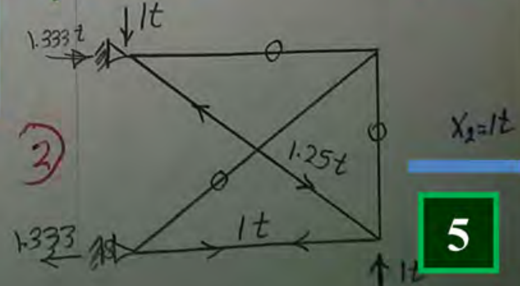
4-5

$$N_q = \begin{bmatrix} 1.333 & 0 \\ 1.333 & 1 \\ 1.0 & 0 \\ -1.667 & 0 \\ -1.667 & -1.25 \end{bmatrix} \quad (4)$$

4



5



6

$$F_u = \frac{1}{10500} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \quad (2)$$

7

$$\Delta_{2\times1} = N_{q\,2\times5}^T \, F_{u\,5\times5} \, N_{p\,5\times1} = \begin{Bmatrix} -0.0114 \\ -0.0089 \end{Bmatrix} \quad (3)$$

8

$$F_{2\times2} = N_{q\,2\times5}^T \, F_{u\,5\times5} \, N_{q\,5\times2} = \begin{bmatrix} 0.0043 & 0.0020 \\ 0.0020 & 0.0026 \end{bmatrix} \quad (3)$$

9

$$X = \begin{Bmatrix} 1.2917 \\ 2.9464 \end{Bmatrix} t \quad (1)$$

(3)

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EX-02 Truss

$J=4$, $m=6$, $r=4$

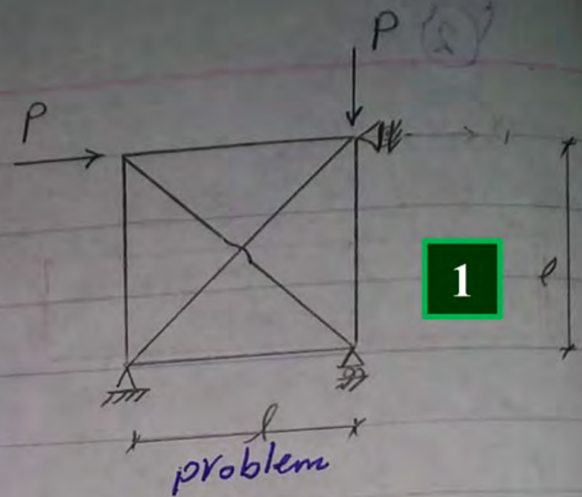
$n = 6+4-2\times 4 = 2$

$$\underline{\Delta}_{(n\times 1)} + \underline{F}_{(n\times n)}\, \underline{X}_{(n\times 1)} = \underline{0}_{(n\times 1)}$$

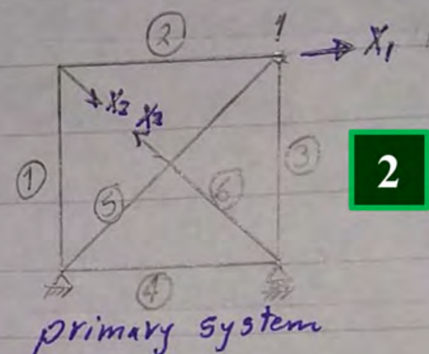
$$\underline{\Delta}_{(2\times 1)} + \underline{F}_{(2\times 2)}\, \underline{X}_{(2\times 1)} = \underline{0}_{(2\times 1)}$$

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

1



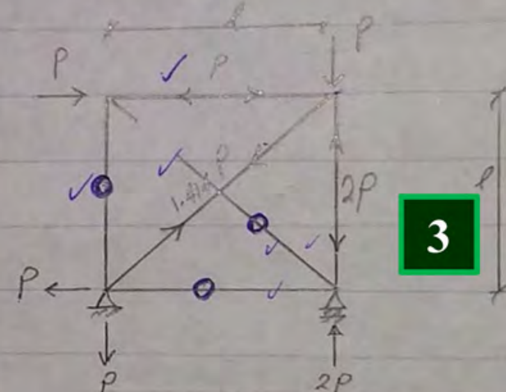
2



To get $\underline{N}_{p\ (m\times 1)\ (6\times 1)}$

$$\underline{N}_{p\,(6\times 1)} = \begin{Bmatrix} 0 \\ -P \\ -2P \\ 0 \\ 1.414P \\ 0 \end{Bmatrix}$$

3

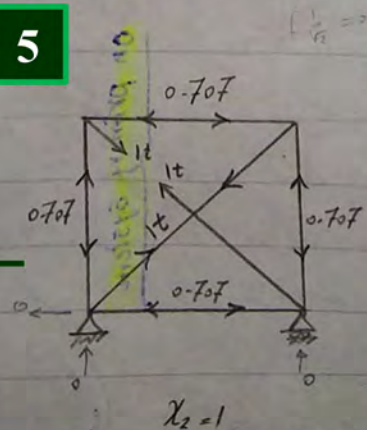


To get $\underline{N}_{q\ (m\times n)\ (6\times 2)}$

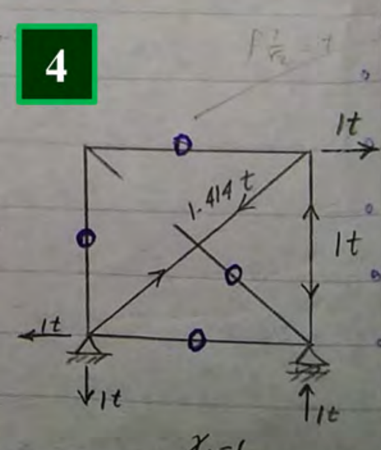
$$\underline{N}_{q\,(6\times 2)} = \begin{bmatrix} 0 & -0.707 \\ 0 & -0.707 \\ -1 & -0.707 \\ 0 & -0.707 \\ 1.414 & 1 \\ 0 & 1 \end{bmatrix}$$

$X_1=1$ $\quad$ $X_2=1$

5



4



6

To get $\underline{F}_{u\,m\times m}$ (6×6)

$$\underline{F}_{u\,6\times6} = \frac{L}{EA}\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1.414&0\\0&0&0&0&0&1.414\end{bmatrix}$$

To get $\Delta_{n\times1}$ (2×1)

$$\underline{\Delta}_{(n\times1)} = \underline{N}_{q\,(n\times m)}^{T}\cdot\underline{F}_{u\,(m\times m)}\cdot\underline{N}_{p\,(m\times1)}$$

7

$$\underline{\Delta}_{(2\times1)} = \underline{N}_{q\,(2\times6)}^{T}\cdot\underline{F}_{u\,(6\times6)}\cdot\underline{N}_{p\,(6\times1)}$$

$$\begin{Bmatrix}\Delta_1\\\Delta_2\end{Bmatrix} = \frac{L}{EA}\begin{bmatrix}0&0&-1&0&1.414&0\\-0.707&-0.707&-.707&-.707&1&1\end{bmatrix}_{2\times6}\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1.414&0\\0&0&0&0&0&1.414\end{bmatrix}_{6\times6}\underline{N}_{p\,6\times1}$$

$\frac{EA}{L}\cdot N_{x_1=1}$

$\frac{EA}{L}\cdot N_{x_2=1}$

$$= \frac{PL}{EA}\begin{bmatrix}0&0&-1&0&2&0\\-0.707&-0.707&-0.707&-0.707&1.414&1.414\end{bmatrix}_{2\times6}\begin{bmatrix}0\\-1\\-2\\0\\1.414\\0\end{bmatrix}_{6\times1}$$

$$\begin{Bmatrix}\Delta_1\\\Delta_2\end{Bmatrix} = \frac{Pl}{EA}\begin{Bmatrix}4.828\\4.121\end{Bmatrix}\begin{matrix}\text{---- } \delta_{01}\\\text{---- } \delta_{02}\end{matrix}$$

To get $\underset{n \times n}{\underset{2 \times 2}{\underline{F}}}$ "the flexibility matrix"

8

$$\underline{F}_{n \times n} = \underset{(n \times m)}{\underline{N_q}^T} \cdot \underset{(m \times m)}{\underline{F_u}} \cdot \underset{(m \times n)}{\underline{N_q}}$$

$$\underline{F}_{(2\times2)} = \underline{N_q}^T_{(2\times6)} \cdot \underline{F_u}_{(6\times6)} \cdot \underline{N_q}_{(6\times2)}$$

δ_{11}, δ_{21} δ_{12}, δ_{22}

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \frac{L}{EA} \begin{bmatrix} 0 & 0 & -1 & 0 & 2 & 0 \\ -0.707 & -0.707 & -0.707 & -0.707 & 1.414 & 1.414 \end{bmatrix} \begin{bmatrix} 0 & -0.707 \\ 0 & -0.707 \\ -1 & -0.707 \\ 0 & -0.707 \\ 1.414 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \frac{L}{EA} \begin{bmatrix} 3.828 & 2.707 \\ 2.707 & 4.828 \end{bmatrix}$$

* Compatibility Condition

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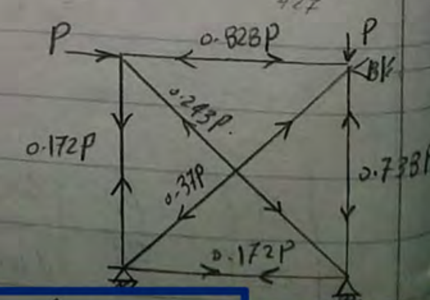
$$\underline{\Delta}_{2\times1} + \underline{F}_{2\times2}\, \underline{X}_{2\times1} = \underline{0}_{2\times1}$$

$$\frac{P\ell}{EA} \begin{Bmatrix} 4.828 \\ 4.121 \end{Bmatrix} + \frac{L}{EA} \begin{bmatrix} 3.828 & 2.707 \\ 2.707 & 4.828 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = -\frac{P\ell \times EA}{EA \times \ell} \frac{1}{(3.828)(4.828) - (2.707)^2} \begin{bmatrix} 4.828 & -2.707 \\ -2.707 & 3.828 \end{bmatrix} \begin{Bmatrix} 4.828 \\ 4.121 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} -1.091\,P \\ -0.243\,P \end{Bmatrix}$$



Final normal forces in the members

$$\underline{N}_{(m\times1)} = \underline{N_P}_{(m\times1)} + \underline{N_q}_{(m\times n)}\, \underline{X}_{(n\times1)}$$

10

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -P \\ -2P \\ 0 \\ 1.414P \end{Bmatrix} + \begin{bmatrix} 0 & -0.707 \\ 0 & -0.707 \\ -1 & -0.707 \\ 0 & -0.707 \\ 1.414 & 1 \end{bmatrix} \begin{Bmatrix} -1.09P \\ -0.243P \end{Bmatrix} = \begin{Bmatrix} 0.172\,P \\ -0.828\,P \\ -0.738\,P \\ 0.172\,P \end{Bmatrix}$$

Annex

Solution for

matlab & SAP for Question of Truss by Force Method

By *DR. Ahmed M. EL-Kholy*

NP =

```
   -3.0000
   -3.6667
   -5.0000
         0
    8.3333
```

NQ =

```
    1.3333         0
    1.3333    1.3333
    1.0000         0
   -1.6667         0
   -1.6667   -1.6667
```

FU =

```
   1.0e-03 *

    0.3810         0         0         0         0
         0    0.3810         0         0         0
         0         0    0.2857         0         0
         0         0         0    0.4762         0
         0         0         0         0    0.4762
```

D =

```
  -0.0114
  -0.0085
```

F =

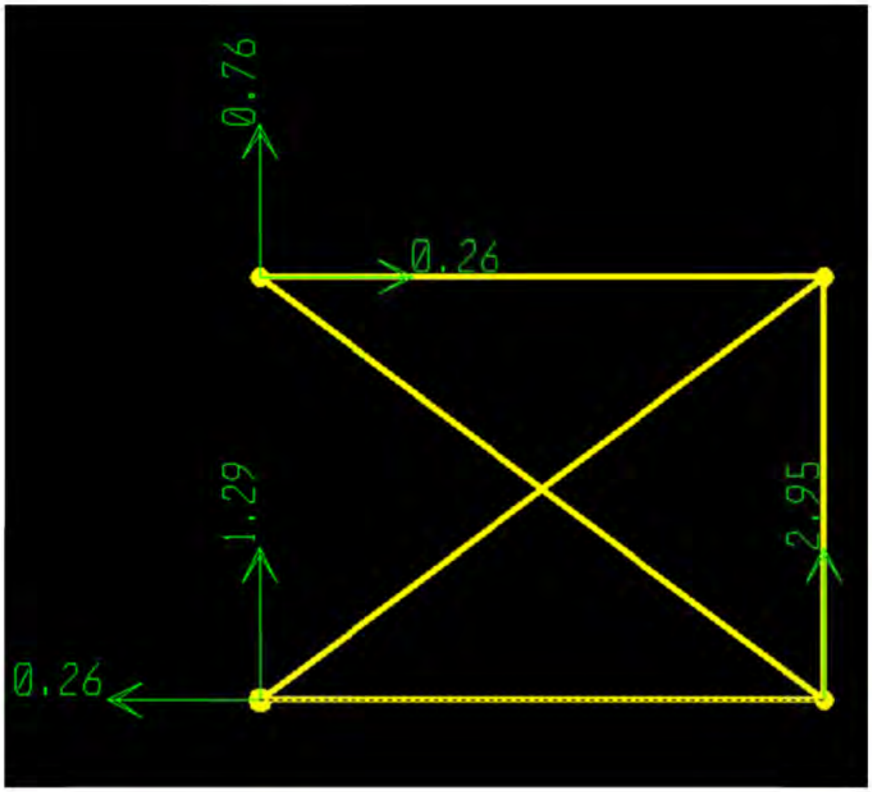
```
   0.0043    0.0020
   0.0020    0.0020
```

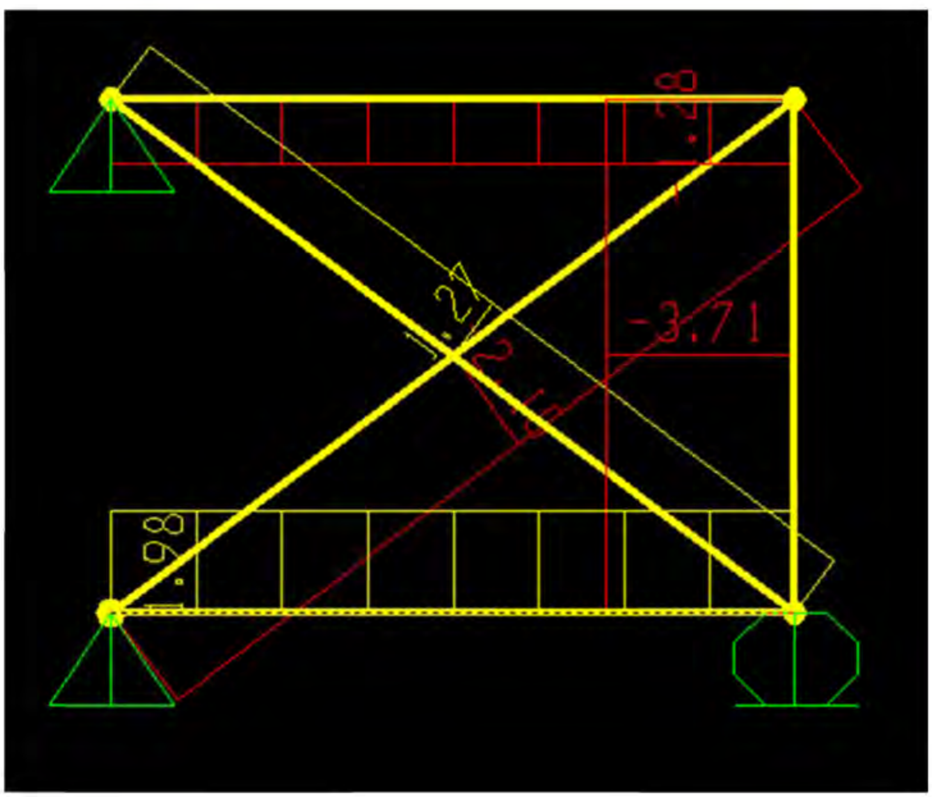
X =

```
   1.2917
   2.9464
```

N =

```
  -1.2778
   1.9841
  -3.7083
  -2.1528
   1.2698
```


0.76
0.26
1.29
2.95
0.26


-1.28
-3.71
1.98